CALCULUS

Finding limits:

Given that x is approaching a real number, use substitution.

1. If the result is a ***real number***, that is the answer.
2. If the result is ***0/0***, then factor or use other algebraic means to eliminate the problem, then substitute.
3. If the result is ***a/0*** where $a \neq 0$, then the answer is infinity. Use the rules for signed numbers to determine (+) or (-).

Given that x is approaching infinity,

1. If the function is a polynomial, consider only the term with the highest degree. Use the rules for signed numbers to determine (+) or (-) infinity.
2. If the function is rational, consider only the terms of highest degree in the numerator and denominator. Simplify and use the previous rules to determine the limit.
3. If the result is ***a/∞***, $a \neq 0$, the answer is zero.

RULES FOR DIFFERENTIATION

Rule 1: **The derivative of a constant function is zero.**

Example 1 Derivatives of Constant Functions

a. $d/dx\,(3) = 0$ because 3 is a constant function

b. If $g(x) = 5$, then $g'(x) = 0$ because g is a constant function. For example, the derivative of g when $x = 4$ is $g'(4) = 0$.

Rule 2: If n is any real number, then $(d/dx)\,(x^n) = nx^{n-1}$ provided x^{n-1} exists.

Example 2 Derivatives of Powers of x

a. By Rule 2, $(d/dx)\,(x^2) = 2x^{2-1} = 2x$

b. If $F(x) = x = x^1$, $F'(x) = (1)\,(x^{1-1}) = (1)\,(x^0) = 1$. Thus the derivative of x with respect to x is 1.

c. If $f(x) = x^{-10}$, then $f'(x) = -10x^{-10-1} = -10x^{-11}$

Example 3 Rewriting Functions in the Form x^n

a. To differentiate y = sq rt (x), we rewrite sq rt (x) as $x^{1/2}$ so that it has the form x^n.

b. Let h(x) = 1/ (*x(sq rt x*)). To apply Rule 2, rewrite $h(x)$ as $h(x) = x^{-3/2}$ so that it has the form x^n.

Rule 3: The derivative of a constant times a function is the constant times the derivative of the function.

Example 4 Differentiating a Constant Times a Function

Solve the following: $g(x) = 5x^3$

Solution: Here *g* is a constant (5) times a function (x^3).

$$(d/dx)\,(5x^3) = 5\,(d/dx)\,(x^3) \qquad \text{(Rule 3)}$$

$$= 5\,(3x^{3-1}) = 15x^2 \qquad \text{(Rule 2)}$$

Rule 4: The derivative of a sum or difference of two functions is the sum or difference of their derivatives.

Differentiate the following: $F(x) = 3x^5 + x^{½}$

$$F'(x) = (d/dx)\,(3x^5) + (d/dx)\,(x^{1/2}) \qquad \text{(Rule 4)}$$

$$= 3\,(d/dx)\,(x^5) + (d/dx)\,(x^{1/2}) \qquad \text{(Rule 3)}$$

$$= 3\,(5x^4) + ½\,x^{-1/2}$$

$$= 15x^4 + 1 / (2\,(\text{sq rt }(x))) \qquad \text{(Rule 2)}$$

Rule 5: The derivative of a product of two functions is the first function times the derivative of the second plus the second function times the derivative of the first.

Example 5: Applying the Product Rule

Solve the following: If $F(x) = (x^2 + 3x)\,(4x + 5)$, find $F'(x)$.

Solution: Consider *F* as a product of two functions

$$F(x) = \underbrace{(x^2 + 3x)}_{f(x)\ \Uparrow}\ \underbrace{(4x + 5)}_{g(x)\ \Uparrow}$$

Now apply the product rule:

$F'(x) = f(x)\ g'(x) + g(x)\ f'(x)$

$= (x^2 + 3x)\ (d/dx)\ (4x + 5) + (4x + 5)\ (d/dx)\ (x^2 + 3x)$

$= (x^2 + 3x)\ (4) + (4x + 5)\ (2x + 3)$

$= 12x^2 + 34x + 15$ (simplifying)

Rule 6: The derivative of a quotient of two functions is the denominator times the derivative of the numerator minus the numerator times the derivative of the denominator, all divided by the denominator squared.

In other words, the derivative of a quotient can be defined as:

$$\frac{\text{(denominator) (deriv. of numerator) – (numerator) (deriv. of denominator)}}{\text{denominator squared}}$$

Example 6: Applying the Quotient Rule

If $F(x) = \dfrac{4x^2 + 3}{2x - 1}$, find $F'(x)$.

Let $f(x) = 4x^2 + 3$ and $g(x) = 2x - 1$. Then use the derivative of a quotient definition.

$$F'(x) = \frac{(2x - 1)\ (8x) - (4x^2 + 3)\ (2)}{(2x - 1)^2}$$

$$= \frac{8x^2 - 8x - 6}{(2x - 1)^2} = \frac{2\ (4x^2 - 4x - 3)}{(2x - 1)^2}$$

Rule 7: (CHAIN RULE) If *y* is a differentiable function of *u* and *u* is a differentiable function of *x*, then *y* is a differentiable function of *x* and

$$(dy/dx) = (dy/du)\,(du/dx)$$

Example 7: Using the Chain Rule

a. If $y = 2u^2 - 3u - 2$ and $u = x^2 + 4$, find dy/dx.

Solution: By the chain rule,

$dy/dx = (dy/du)(du/dx) = d/du\,(2u^2 - 3u - 2)\;\; d/dx\,(x^2 + 4)$

$= (4u - 3)\,(2x)$

The answer can be written in terms of *x* alone by replacing *u* by $x^2 + 4$.

$$dy/dx = [\,4(x^2 + 4) - 3]\,(2x) = [4x^2 + 13]\,(2x) = 8x^3 + 26x$$

b. If y = sq rt (w) and $w = 7 - t^3$, find dy/dt.

Solution: Here *y* is a function of *w* and *w* is a function of *t*, so view *y* as a function of *t*. By the chain rule,

$dy/dt = (dy/dw)\,(dw/dt) = d/dw$ (sq rt w) $d/dt\,(7 - t^3)$

$= (½\, w^{-1/2})\,(-3t^2) = (1/(2$ sq rt $w))\;(-3t^2)$

$= -3t^2 / (2$ sq rt $w) = -3t^2 / (2$ sq rt $(7 - t^3))$

Rule Eight: (POWER RULE) If *u* is a differentiable function of *x* and *n* is any real number, then

$$d/dx\,(u^n) = nu^{n-1}\;\; du/dx$$

Example 8: Using the Power Rule

If $y = (x^3 - 1)^7$, find y'.

Solution: Since y is a function of x, the power rule applies. Letting

$u(x) = x^3 - 1$ and $n = 7$,

$y' = n\,[u(x)]^{n-1}\ u'(x)$

$= 7\,(x^3 - 1)^{7-1}\ d/dx\,(x^3 - 1)$

$= 7\,(x^3 - 1)^6\,(3x^2) = 21x^2\,(x^3 - 1)^6$