Volumes of Solids of Revolution: Disk/Washer and Shell Methods

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For problems 1 - 2, let R be the region bounded by the given curves. Sketch R. If R is revolved about the x-axis, find the volume of the solid of revolution (a) by the disk/washer method, and (b) by the shell method. Show that the results are the same.

1. $y = x^2$, $y = 2x$

2. $y = \sqrt{x}$, $y = x^3$

For problems 3 - 4, let R be the region bounded by the given curves. Sketch R. If R is revolved about the y-axis, find the volume of the solid of revolution (a) by the disk/washer method, and (b) by the shell method. Show that the results are the same.

3. $y = x^2$, $x = y^2$

4. $y = 4 - x^2$, $y = 1$

Hints about set-up:

1. a) $V = \pi \int_0^2 \left[(2x)^2 - \left(x^2\right)^2 \right] dx$ b) $V = 2\pi \int_0^4 y \left[\left(\sqrt{y}\right) - \left(\frac{1}{2} y\right) \right] dy$

2. a) $V = \pi \int_0^1 \left[\left(\sqrt{x}\right)^2 - \left(x^3\right)^2 \right] dx$ b) $V = 2\pi \int_0^1 y \left[\left(\sqrt[3]{y}\right) - \left(y^2\right) \right] dy$

3. a) $V = \pi \int_0^1 \left[\left(\sqrt{y}\right)^2 - \left(y^2\right)^2 \right] dy$ b) $V = 2\pi \int_0^1 x \left[\left(\sqrt{x}\right) - \left(x^2\right) \right] dx$

4. a) $V = \pi \int_1^4 \left[\left(\sqrt{4-y}\right)^2 \right] dy$ b) $V = 2\pi \int_0^{\sqrt{3}} x \left[\left(4 - x^2\right) - (1) \right] dx$

Answers:

1. $\frac{64\pi}{15}$

2. $\frac{5\pi}{14}$

3. $\frac{3\pi}{10}$

4. $\frac{9\pi}{2}$