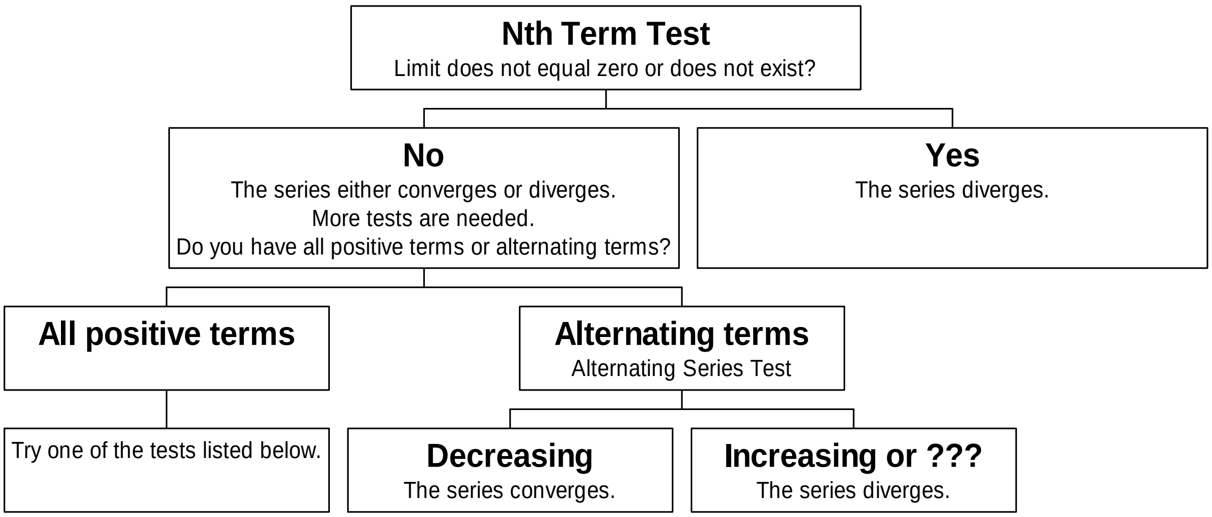
Tests for Series Convergence
Jefferson Davis Learning Center, Sandra Peterson

Suppose you have a series that looks like this: $\sum_{n=1}^{\infty} a_n$

Begin by finding: $\lim_{n \to \infty} a_n \neq 0$



If your series has all POSITIVE terms, then you should try one of the following tests:

I. **The Geometric Series**

$\sum_{n=1}^{\infty} ar^n$ where a and r are constants.

A. If $|r| \geq 1$, then the series diverges.

B. If $|r| < 1$, then the series converges. The sum is $S = \frac{a}{1-r}$.

II. **The P-Series**

$\sum_{k=1}^{\infty} \frac{1}{k^p}$ where p is a constant.

A. If $0 < p \leq 1$, then the series diverges.

B. If $p > 1$, then the series converges.

III. **The Comparison Test** (for positive terms only)

Let $\sum a_k$ and $\sum b_k$ be two series such that $a_n \leq b_n$ for all n.

A. If $\sum b_k$ ("bigger" series) converges, then $\sum a_k$ ("smaller" series) converges.

B. If $\sum a_k$ ("smaller" series) diverges, then $\sum b_k$ ("bigger" series) diverges.

IV. **The Limit Comparison Test** (for positive terms only)

Let $\sum a_k$ and $\sum b_k$ be two series. And let $\rho = \lim_{k \to +\infty} \frac{a_k}{b_k}$.

If ρ is finite and $\rho > 0$, then the series both:

A. converge (smaller than a convergent series converges)

B. OR diverge (bigger than a divergent series diverges)

V. **The Ratio Test** (for positive terms only)

Let $\sum u_k$ be a series and suppose that $\rho = \lim_{k \to +\infty} \frac{u_{k+1}}{u_k}$.

A. If $\rho < 1$, the series converges.

B. If $\rho > 1$ or if $\rho = +\infty$, then the series diverges.

C. If $\rho = 1$, then the series may converge or diverge. You must use another test.

VI. **The Root Test** (for positive terms only)

Let $\sum u_k$ be a series and suppose that $\rho = \lim_{k \to +\infty} \sqrt[k]{u_k} = \lim_{k \to +\infty} (u_k)^{i/k}$.

A. If $\rho < 1$, the series converges.

B. If $\rho > 1$ or if $\rho = +\infty$, then the series diverges.

C. If $\rho = 1$, then the series may converge or diverge. You must use another test.

VII. **The Integral Test** (for positive terms only)

Let $\sum u_k$ be a series and suppose that $f(x)$ is the function that results when k is replaced with x in the formula for u_k. If $f(x)$ is decreasing and continuous on the interval $[a, +\infty)$, then $\sum_{k=1}^{\infty} u_k$ and $\int_a^{+\infty} f(x)\,dx = \lim_{l \to +\infty} \int_a^{l} f(x)\,dx$ both converge or both diverge.